Correspondence

Data Performance in Burst Switching When the Voice Silence Periods Have a Hyperexponential Distribution

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Abstract—**The performance of data in burst switching has been analyzed in previous work with a fluid approximation of the data traffic. This study extends the previous model to the case where the silence interval between talkspurts has a hyperexponential, rather than an exponential, distribution. It is shown that data performance is extremely sensitive to the variance of the silence interval, and that, for empirical talkspurt and silence distributions, this model provides a vast improvement on models which assume that both types of intervals are exponentially distributed.**

I. Introduction

In a performance analysis of data traffic in burst switching, O'Reilly [1], [2], assumed that both the talkspurt and silence intervals in a voice conversation are exponentially distributed. This assumption of exponentiality has also been made by all other analytic models of DSI systems encountered in the literature. The assumption is generally necessary for analytic tractability. However, the use of a fluid approximation in the analysis of the data performance in an integrated system provides not only accurate estimates of the data performance but is computationally very fast; thus, extension of the model in [1], [2] to more complex speaker models is feasible.

A conclusion of the study in [1] was that a two-state Markov model for a single speaker, which implies approximating both the talkspurts and silence period distributions as exponential, was inadequate for empirical voice conversations. A comparison to empirical distributions obtained for talkspurts and silences, such as found by Yatsuzuka [3], shows that an exponential approximation for a silence period distribution is particularly inadequate. Although the talkspurt coefficient of variation in [3] is slightly less than one, making the exponential assumption relatively easy to justify, the coefficient of variation of the silence distribution is almost three.

In this paper, we develop a three-state model for the voice process, with a hyperexponential distribution for the silence interval length, such that the first two moments of the silence interval can be matched with that of the Yatsuzuka or any other empirical silence distribution.

The resulting two-dimensional voice process for a fixed number of voice sources is developed and analyzed. A fluid-flow approximation for the data process is then used to find the data queue length distribution. Due to the bivariate nature of the voice process, the solution of the data queue length distribution involves the solution of a multidimensional differential equation of size $(S+1)(S+2)/2$ where S is the number of voice sources. Thus, the computational complexity is of the order S^2, whereas the exponential model in [2] has complexity of the order S. Although the use of a more complex speaker model necessarily increases the complexity, the model is nonetheless computationally feasible because of the fluid approximation used in the data traffic analysis.

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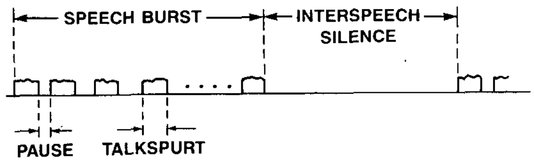

Fig. 1. Voice process.

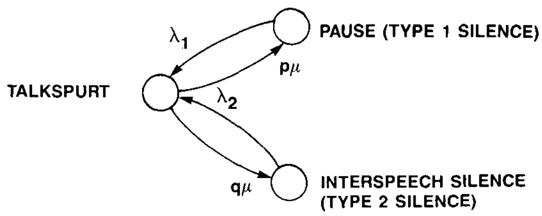

Fig. 2. Three-state Markov model for a single speaker.

II. Model Formulation and Assumptions

A large coefficient of variation of the silence period distribution has been obtained in a number of empirical studies [3], [4], of highly sensitive speech activity detectors. This can be explained intuitively as follows. In a typical two-way conversation, we observe two types of silences:

1) Frequent short pauses occurring in speech when a speaker momentarily hesitates or due to the occurrence of a stop consonant (Type 1 silence).

2) Much longer silences occurring when a speaker stops to listen to the other party (Type 2 silence).

Such a voice process is shown in Fig. 1. We define a speech burst as the interval over which one speaker holds the conversation, and an interspeech silence as that timespan when the other speaker speaks. As shown in the figure, a speech burst itself consists of alternating talkspurts and pauses.

If we assume that the talkspurt, pause, and interspeech silence periods are each exponentially distributed with parameters μ, λ_1, λ_2, respectively, then the voice process of a single speaker can be modeled as a three-state Markov chain as shown in Fig. 2. If we further assume that, on the average, speech bursts of both parties in a conversation are equally long and that there is no "dead" time between speakers, then the probability p that a talkspurt is followed by a pause is given approximately by

$$p=(n-1)/n$$

where n is the average number of talkspurts in a speech burst. Clearly, n is given by

$$n=(1/\lambda_2)/(1/\mu+1/\lambda_1).$$

Thus,

$$p=1-\lambda_2(1/\mu+1/\lambda_1).$$

The distribution of the combined pause and silence periods is the two-stage hyperexponential distribution with probability density function

$$p\lambda_1 e^{-\lambda_1 t} - q\lambda_2 e^{-\lambda_2 t}, \; t>0$$

where $q = 1 - p$. The mean and variance of this distribution are given, respectively, by

$$p/\lambda_1 + q/\lambda_2,$$

and

$$2[p/\lambda_1^2 + q/\lambda_2^2] - [p/\lambda_1 + q/\lambda_2]^2.$$

It is well known that the coefficient of variation of a hyperexponential is greater than 1. Thus, we can match the mean and variance of the hyperexponential to that of the Yatsuzuka or any other empirical silence distribution by a suitable choice of λ_1 and λ_2. These values of λ_1, λ_2, and thus p, are unique for a given mean and variance.

For the Yatsuzuka silence distribution the mean and standard deviation are 480 and 1379 ms, respectively. These values are matched by a hyperexponential of pause and interspeech means 105.6 ms and 2.712 s, respectively. With an average talkspurt length of 284 ms, these latter values give $p = 8.8563$ (and $q = 0.1437$).

The activity factor for a single voice source p', defined as the probability that a voice source is in talkspurt, is given by

$$p' = \lambda/(\lambda + \mu) \tag{1}$$

where $1/\lambda$, the effective average length of a silence period, is given by the weighted sum of the interspeech silence and pause periods

$$1/\lambda = p/\lambda_1 + q/\lambda_2. \tag{2}$$

Substituting (2) into (1) gives

$$p' = (1 + p\mu/\lambda_1 + q\mu/\lambda_2)^{-1}.$$

We now develop a model[1] for a voice process with S off-hook voice sources, to be transmitted over a link with a total of $(c+v)$ TDM channels with c reserved for data and v shared between voice and data. We assume that voice talkspurts have preemptive priority over data messages. This has been shown in [2] to be a valid assumption for burst switching with typical talkspurt lengths of 280 ms and data message lengths of less than 20 ms.

Define $A(t)$ as the number of sources in talkspurt at time t and $D(t)$ as the deterministic process (following the fluid approximation) approximating the number of data messages in the queue. The data are assumed to arrive at a rate of δ (messages/s) and to be serviced at a rate η (messages/s) per channel. We define the channelized data utilization ρ_d as the ratio δ/η.

Let $B(t)$ be the number of voice calls in a silence period (of either Type 1 or Type 2). Thus, $B(t) = S - A(t)$. If $B(t) = k$, then $c + \max\{0, v - (S-k)\} = c + [v - (S-k)]^+$ channels are available for data transmission, and thus the service rate is $\{c + [v - (S-k)]^+\}\eta$. Let the difference between the data arrival and service rates be represented as r_k:

$$r_k = \delta - \{c + [v - (S-k)]^+\}\eta \tag{3}$$

and we let N be such that

$$r_S < r_{S-1} < \cdots < r_{S-N} \le 0 < r_{S-N-1} < \cdots < r_{S-v}.$$

Note that since k represents the number of sources in silence (rather than in talkspurt), the definition of r_k in (3) and their subsequent sequencing order are both reversed from the corresponding definition and ordering in [1]. The parameter N has, however, the same interpretation as before.

[1] The following model closely follows [1], [2] and unless otherwise stated, the assumptions made and notation used are the same.

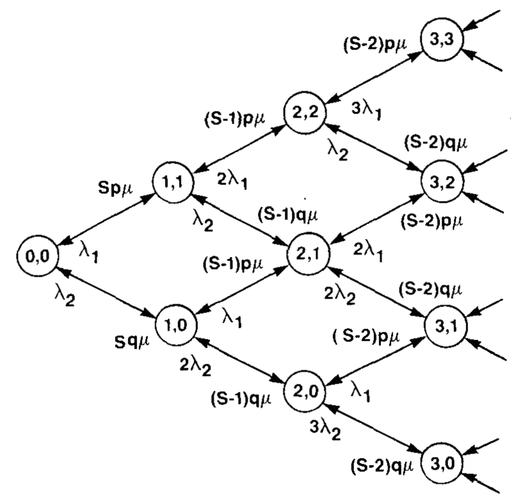

Fig. 3. Silent-period process.

III. Analysis of Voice Process

Having assumed preemption of data messages by voice, we can analyze the voice process independently of the data traffic. The silent-period process is defined as $\{(B(t), B1(t), t > 0\}$ where $B(t)$ is the number of voice calls in silence period (of either Type 1 or 2) at time t, and $B1(t)$ is the number of Type 1 only (at t); therefore, the number of Type 2 is $B(t) - B1(t)$. The silent-period process can be modeled as a bivariate birth-death process, as shown in Fig. 3. In general, the transition rates are given by

$$q_v((k, i), (k+1, i+1)) = (S-k)p\mu$$

$$q_v((k, i), (k+1, i)) = (S-k)q\mu$$

$$q_v((k, i), (k-1, i)) = (k-i)\lambda_2$$

$$q_v((k, i), (k-1, i-1)) = i\lambda_1$$

where we define $q_v((k, i), (k', i'))$ as the transition rate from state (k, i) to (k', i'). For the boundary states, of course, some of these rates will be zero. Let $\theta(k, i)$ represent the equilibrium distribution of this process, i.e.,

$$\theta(k, i) = \lim_{t\to\infty} \text{Prob}\,\{B(t) = k, B1(t) = i\}.$$

The equilibrium distribution has product form and can be shown to be

$$\theta(k, i) = \binom{S}{i}\binom{S-i}{k-i}(p\mu/\lambda_1)^i (q\mu/\lambda_2)^{k-i}(p')^S.$$

From this, it is easy to show that the probability that k sources are silent, $\theta(k)$, is given by

$$\theta(k) = \sum_{i=0}^{k} \theta(k, i) = \binom{S}{k}(p')^{S-k}(q')^k, \qquad k = 0, \cdots, S$$

so that, as expected, the distribution of the number of talkspurts is binomial.

When the number of talkspurts exceeds the number of transmission channels v, the talkspurts which began when all channels were busy are "frozen out," and front-end clipping of the newest talkspurts takes place. Thus, the average cutout, i.e., the average fraction of a talkspurt lost due to freezeout, is given by

$$\phi = \frac{1}{Sp'} \sum_{k=v+1}^{S} (k-v) \binom{S}{k} (p')^k (1-p')^{S-k}.$$

In fact, Weinstein has shown [5] that the average cutout is independent of the talkspurt and silence period distributions. The maximum stationary data utilization ρ_{max}, which is also independent of the talkspurt and silence period distributions, is given by

$$\rho_{max} = c + v - Sp(1-\phi).$$

IV. Analysis of Data Performance

In order to find the equilibrium distribution for the data queue length we use the standard procedure of writing down the forward equations. Let $p(x, k, i, t)$ denote the joint probability of having voice in the state $\{B(t) = k, B1(t) = i\}$ and x data messages queued at time t. Consider two cases:

1) No talkspurts are being clipped, i.e., $k = S - v + 1, \cdots, S$. If the voice state remains in state k over a small interval of time $(t, t + \Delta t)$, then the data queue changes by an amount of $r_k \Delta t$ in that time provided that the amount of data present at time t is greater than zero.

2) One or more talkspurts are being clipped, i.e., $k = 0, \cdots, S - v$. Now the data queue increases by an amount $r_{S-v}\Delta t$ in the interval $(t, t + \Delta t)$.

Consequently, treating x as a continuous variable, we get the following two sets of forward equations, for $x > 0$, describing the flow between states. For $k = S - v + 1, \cdots, S - 1$, we get

$$\begin{aligned} p(x, k, i, t+\Delta t) &= p(x - r_k \Delta t, k, i, t)(1 - \{(S-k)\mu + i\lambda_1 + (k-i)\lambda_2\}\Delta t] \\ &+ p(x - r_{k-1}\Delta t, k-1, i-1, t) \cdot (S-k+1)p\mu\Delta t \\ &+ p(x - r_{k-1}\Delta t, k-1, i, t) \cdot (S-k+1)q\mu\Delta t \\ &+ p(x - r_{k+1}\Delta t, k+1, i, t) \cdot (k+1-i)\lambda_2\Delta t \\ &+ p(x - r_{k+1}\Delta t, k+1, i+1, t) \cdot (i+1)\lambda_1\Delta t \\ &+ o(\Delta t) \qquad i = 0, \cdots, k. \end{aligned} \tag{4}$$

For the states $k = 1, \cdots, S - v - 1$, we get

$$\begin{aligned} p(x, k, i, t+\Delta t) &= p(x - r_{S-v}\Delta t, k, i, t)[1 - \{(S-k)\mu + i\lambda_1 + (k-i)\lambda_2\}\Delta t] \\ &+ p(x - r_{S-v}\Delta t, k-1, i-1, t) \cdot (S-k+1)p\mu\Delta t \\ &+ p(x - r_{S-v}\Delta t, k-1, i, t) \cdot (S-k+1)q\mu\Delta t \\ &+ p(x - r_{S-v}\Delta t, k+1, i, t) \cdot (k+1-i)\lambda_2\Delta t \\ &+ p(x - r_{S-v}\Delta t, k+1, i+1, t) \cdot (i+1)\lambda_1\Delta t \\ &+ o(\Delta t) \qquad i = 0, \cdots, k. \end{aligned} \tag{5}$$

The boundary equations for $k = 0, S - v$, and S are special cases of (4) and (5).

We now use the same solution method as detailed in [1] to obtain the equilibrium probabilities. With

$$p_{k,i}(x) = \lim_{t\to\infty} p(x, k, i, t)$$

and with $p'_{k,i}(x)$ the derivative with respect to x, then for all nonzero r_k the resulting equations for $k = S - v + 1, \cdots, S - 1$ become

$$\begin{aligned} p'_{k,i}(x) &= -\{(S-k)\mu + i\lambda_1 + (k-i)\lambda_2\} p_{k,i}(x)/r_k \\ &+ p_{k-1,i-1}(x) \cdot (S-k+1)p\mu/r_k \\ &+ p_{k-1,i}(x)(S-k+1)q\mu/r_k \\ &+ p_{k+1,i}(x) \cdot (k+1-i)\lambda_2/r_k \\ &+ p_{k+1,i+1}(x)(i+1)\lambda_1/r_k \qquad i = 0, \cdots, k. \end{aligned}$$

A similar set of equations is written for $k = 1, \cdots, S - v - 1$ and for the boundary states $k = 0, S - v$, and S. Note that x is continuous and $p_{k,i}(x)$ is valid only for $x > 0$. Let $\pi_{k,i}$ denote the probability of having $x = 0$ and voice being in state (k, i). For the states $k = S - N, \cdots, S$, $\pi_{k,i}$ has a nonzero probability, whereas $\pi_{k,i} = 0$ for $k < S - N$.

The set of equations for $p'_{k,i}(t)$, $k = 0, \cdots, S$ and $i \le k$, can be written in matrix form by first transforming the elements of the two-dimensional density $p_{k,i}(x)$ into a one-dimensional function $p_j(x)$. We use a nonlinear mapping function f, such that $j = f(k, i)$, which gives a one-to-one mapping between the states $\{(k, i); k = 0, \cdots, S; i = 0, \cdots, k\}$ and $\{j; j = 1, \cdots, J\}$ where J, the number of states in the silent-period process, equals $(S + 1)(S + 2)/2$. Since f only determines the ordering of the states in the matrix, it does not affect the solution of the system. Choosing an appropriate f makes the matrix less scattered and the numerical solution more efficient.

Thus, the set of differential equations may now be written in matrix form as

$$P'(x) = -AP(x) \qquad x > 0$$

where $P(x)$ is a J-column vector representing the transpose of $[p_1(x) \cdots p_J(x)]$ and A is a square matrix of dimension J. Finding the strictly positive eigenvalues of A, the corresponding eigenvectors, and using appropriate boundary conditions (the solution technique is described in [1]), we can evaluate the steady-state probability that the voice is in state j and that x data messages are queued (for $j = 1, \cdots, J$ and $x > 0$). Then we can find the probability density function $p(x)$ of the data queue length from

$$p(x) = \sum_{j=0}^{J} p_j(x) \qquad x > 0.$$

Subsequently, we can evaluate π_j for all j such that $k \ge S - N$, using the boundary conditions and $p_j(x)$. This leads directly to the probability π of the system being empty. Other performance measures directly follow from $p(x)$.

V. Performance Studies

The sensitivity of data performance to the variance of the hyperexponentially distributed silence interval has been studied for a number of systems. Fig. 4 shows the average data queueing delay as a function of data utilization for systems with one speaker ($S = 1$), one shared channel ($v = 1$), and no channels reserved for data ($c = 0$). Other system parameters are listed in the figure. As the coefficient of variation of the silence distribution is increased from 1 (exponential) to 2.87, and then 4.82, the data performance degrades consistently with increasing variation in the silence interval lengths. From these results and similar results for larger systems, we conclude that the data performance is extremely sensitive to the variance of the silence interval; without a doubt, the same is true of its sensitivity to the variance of the talkspurt length.

Fig. 5 shows similar results for a larger system: eight shared channels, no reserved channels, and 13 off-hook voice sources. Talkspurts and silence intervals have an average length, respectively, of 284 and 480 ms. Data messages have

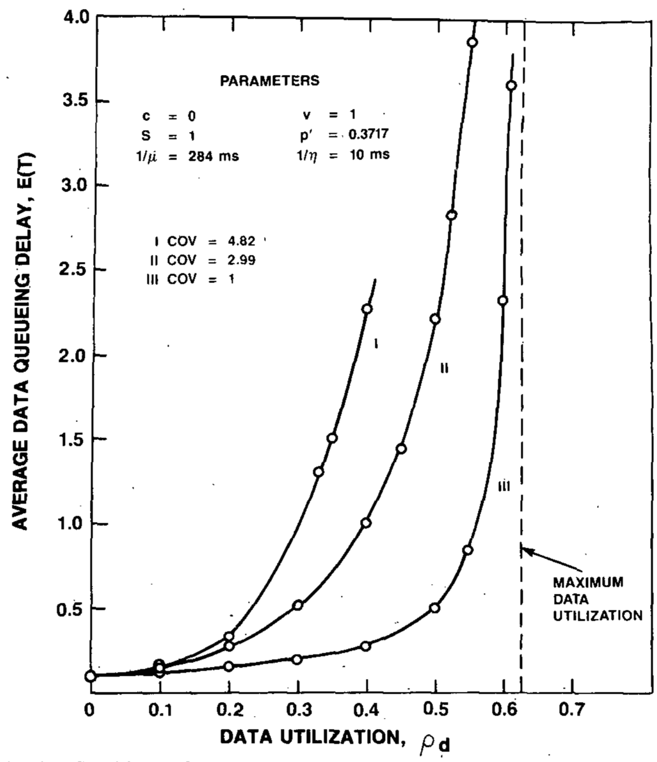

Fig. 4. Sensitivity of data performance to the coefficient of variation (COV) of the hyperexponential voice silence distribution.

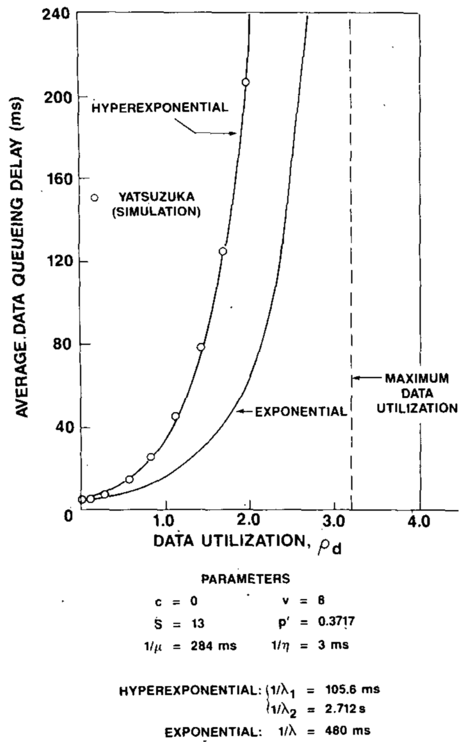

Fig. 5. Comparison of data performance for different talkspurt/silence distributions.

an average length of 3 ms. Results are shown here for three different distributions of the talkspurt and silence intervals: 1) both talkspurts and silences exponentially distributed; 2) the talkspurts exponentially distributed and the silences hyperexponential with the parameters chosen to match the first two moments of the Yatsuzuka distribution; and 3) the Yatsuzuka distribution itself. The latter results are obtained using a simulation model of a burst-switched link. (Recall that in burst switching the voice priority is not preemptive.) We note that the difference between the hyperexponential and exponential is substantial while those based on the empirical distribution match the hyperexponential curve very well. A similar correspondence between the simulation results and results of the hyperexponential model has been found for systems of smaller and somewhat larger capacity.

This agreement between the simulation and hyperexponential models occurs despite the following divergences of the analytic model from reality:

- the assumption of an exponential distribution for the talkspurt distribution,
- the fact that the actual silence interval distribution is not hyperexponential,
- the approximations inherent in the analytic model, namely, the voice preemption as assumption and the fluid approximation.

Clearly, the effects of many of these assumptions are not very significant; also they tend, to a certain extent at least, to cancel each other out. Thus, it can be concluded that the hyperexponential assumption gives a very satisfactory approximation as far as average data performance is concerned.

Acknowledgment

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Correction to "Statistics of Rayleigh Backscatter from a Single-Mode Fiber"

P. HEALEY

In (1) of the above paper,[1] the delta-function operator was inadvertently omitted. The correct equation is given below.

$$h(z)=U(z)\exp(-2\alpha z)\sum_k p_k a_k \delta(z-z_k).$$

Equation (33) of the above paper[1] was not labeled; it is the equation directly above (34).

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